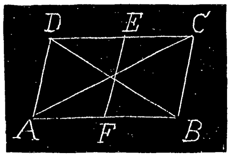


$$\Sigma my^2=\rho\sin^3\beta\int_{-\frac{1}{2}c}^{\frac{1}{2}c}\int_0^c y^2\,dxdy=\tfrac{1}{3}mc^2\sin^2\beta.$$

$$\therefore\ \tan2\theta=\frac{4\sin2\beta}{1+4\cos2\beta}.$$

But $2\theta=\beta+2\varphi$. $\therefore\ \tan2\varphi=\frac{3}{5}\tan\beta$.

$A\cos^2\theta+\beta\sin^2\theta=\frac{1}{3}m(c^2+4c^2\cos^2\beta)$, $A\sin^2\theta+\beta\cos^2\theta=\frac{1}{3}mc^2\sin^2\beta$.

$\therefore\ A=\frac{1}{24}mc^2[5+\sqrt{(17+8\cos2\beta)}]$. $B=\frac{1}{24}mc^2[5-\sqrt{(17+8\cos2\beta)}]$.

$$c^2=a^2+b^2,\ \cos\beta=\frac{a^2-b^2}{a^2+b^2},\ \cos2\beta=\frac{(a^2+b^2)^2-8a^2b^2}{(a^2+b^2)^2}.$$

$\therefore\ A=\frac{1}{24}m\{5a^2+5b^2+\sqrt{[25(a^2+b^2)-64a^2b^2]}\}$.

$B=\frac{1}{24}m\{5a^2+b^2-\sqrt{[25(a^2+b^2)-64a^2b^2]}\}$.

106. Proposed by J. E. CRAIG, A. B., New Germantown, N. J.

The centers of the two wheels of a bicycle are three feet apart.

(1) If a rider wishes the rear wheel to trace a circle 14 feet in diameter, what must be the diameter of the circle traced by the front wheel?

(2) If the rider weighs 120 pounds, and his center of gravity is 3 feet from the ground, at what angle must he lean to make one revolution of the circle every 3 seconds?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(1) The line that is tangent to the hind wheel's path passes through the center of the front wheel, caused by the rigid frame work.

$\therefore\ R=\sqrt{(9+49)}=\sqrt{58}=7.615773$ feet.

$2R$=diameter traced by front wheel=15.231546 feet.

(2) Since the man must be in equilibrium we do not use his weight, and we may regard him as moving around in the plane of the track. Suppose he is midway between the wheels.

Let θ be his inclination to the vertical, g=gravity, $f=v^2/r$=centrifugal force. Then $g\sin\theta$=component of gravity perpendicular to man's direction, $f\cos\theta$=component of centrifugal force perpendicular to same direction. When in equilibrium these forces equalize each other.

$\therefore\ g\sin\theta=f\cos\theta=(v^2/r)\cos\theta$.

$\therefore\ \tan\theta=v^2/gr$.

$r=\sqrt{(49+\frac{9}{4})}=7.1589$ feet. $v=14.3178\pi/3$ feet$=14.9936$ feet per second.

$$\therefore\ \tan\theta=\frac{(14.9936)^2}{32\frac{1}{6}\times7.1589}=.97625.$$

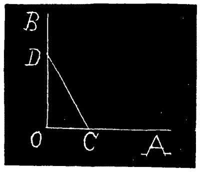
$\therefore\ \theta=44^\circ\ 18'\ 41''$.

107. Proposed by M. E. GRABER, Student, Heidelberg University, Tiffin, O.

Two particles attracting each other inversely as the square of their distances apart, are constrained to move in straight lines which intersect each other at right angles. How long will it take for the particles to meet and how far does each particle *move?*

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $OA=a$, $OB=b$, $OD=u$, $OC=x$, m=mass of particle at C, n=mass of particle at D; A, B the positions of the particles at the beginning of motion; C, D their positions at any time t; v, v_1 their velocities at C, D. Then the equations of motion are



for C, $\dfrac{d^2x}{dt^2}=\dfrac{nx}{(x^2+u^2)^{\frac{3}{2}}}=F$, since $s=x$,

for D, $\dfrac{d^2u}{dt^2}=\dfrac{mu}{(x^2+u^2)^{\frac{3}{2}}}=f$, since $\sigma=u$. But $v=Ft$, $v_1=ft$.

$\therefore\ v_1=\dfrac{muv}{nx}$. Also $\dfrac{a-x}{v}=\dfrac{b-u}{v_1}=\dfrac{(b-u)nx}{muv}$.

$\therefore\ u=\dfrac{bnx}{am-mx+nx}\ ..(1).\quad \therefore\ \dfrac{d^2x}{dt^2}=\dfrac{n(am-mx+nx)^3}{x^2[a^2n^2+(am-mx+nx)^2]^{\frac{3}{2}}}\ ..(2).$

From (1), $u=0$ when $x=0$. Therefore, the particles both arrive at O at the same time. Hence C moves over distance a, and D moves over distance b before they meet. The time is found by integrating (if possible) (2) twice.

If $m=n$, $\dfrac{d^2x}{dt^2}=\dfrac{a^3m}{x^2\sqrt{[(a^2+b^2)^3]}}$.

$$\therefore\ \left(\frac{dx}{dt}\right)^2=\frac{2a^3m}{(a^2+b^2)^{\frac{3}{2}}}\left(\frac{1}{x}-\frac{1}{a}\right)=\frac{2a^2m}{(a^2+b^2)^{\frac{3}{2}}}\cdot\frac{a-x}{x}.$$

$$\therefore\ t=\frac{(a^2+b^2)^{\frac{3}{4}}}{a\sqrt{2m}}\int_0^a\sqrt{\frac{x}{a-x}}\,dx=\frac{\pi(a^2+b^2)^{\frac{3}{4}}}{2\sqrt{(2m)}}.$$

If $a=b$ and m=unity, $t=\pi(\frac{1}{2}a^2)^{\frac{3}{4}}$.

AVERAGE AND PROBABILITY.

92. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

A circular field, radius r, is divided into four *equal* parts, by concentric circles and three concentric rings. From the center of this field are fired *at random*, and with such a velocity as not to produce a range greater than the radius of the field, m=1000 projectiles of the *same* kind. How many projectiles should have fallen into each one of these four equal parts of the field?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The range=$(v^2/g)\sin2\theta$, where θ=angle of elevation. Greatest range $=v^2/g=r$.